Written Exam at the Department of Economics summer 2020

Industrial Organization

Final Exam

29 May 2020

(5-hour open book exam)

Answers only in English.

Upload your answers in Digital Exam as one pdf. file (including appendices) and name your pdf with your examination number only, e.g., 12.pdf or 127.pdf

This exam question consists of 4 pages in total including this front page.

This exam has been changed from a written Peter Bangsvej exam to a take-home exam with helping aids. Please read the following text carefully in order to avoid exam cheating.

Be careful not to cheat at exams!
You cheat at an exam, if you during the exam:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text. This also applies to text from old grading instructions.
- Make your exam answers available for other students to use during the exam
- Communicate with or otherwise receive help from other people
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Use parts of a paper/exam answer that you have submitted before and received a passed grade for without making use of source referencing (self plagiarism)

You can read more about the rules on exam cheating on the study information pages in KUnet and in the common part of the curriculum section 4.12.

Exam cheating is always sanctioned with a warning and dispelling from the exam. In most cases, the student is also expelled from the university for one semester.

Attempt both questions.
Explain all the steps of your analysis and define any new notation that you use.
Show all the calculations that your analysis relies on.

Question 1: Cooperation and advertising

Consider a market in which two firms (indexed by $i = 1, 2$) are competing. How much each firm is selling, and therefore how large profits the firms earn, depend on how much the firms advertise. In particular, what matters for each firm's sales is how much advertising the two firms do *in aggregate*—because an advert for firm 1's product effectively informs the consumers also about firm 2's product (and vice versa). We formalize this idea by specifying the following (reduced-form) profit function for firm i:

$$\pi_i(x_1, x_2) = x_1 + x_2 - \frac{x_i^2}{2a_i}, \tag{1}$$

where $x_i \geq 0$ is firm i's advertising level and $a_i > 0$ is a firm-specific efficiency parameter. We thus abstract from the firms' choices of a price or a quantity, and only consider the impact of the advertising levels on the firm profits (this is the sense in which (1) is a reduced-form profit function).

To start with, we assume that the two firms interact in a simultaneous-move, one-shot game. Specifically, the two firms simultaneously choose their own advertising level, x_i, with the objective of maximizing the profit function in (1).

(a) Solve for the Nash equilibrium of the advertising game described above.

A pair of advertising levels is said to be *Pareto efficient* if there exists no other pair of advertising levels that makes neither firm worse off and at least one firm strictly better off.

Formally, (x_1, x_2) is Pareto efficient if there exists no (x_1', x_2') such that $\pi_i(x_1', x_2') \geq \pi_i(x_1, x_2)$ for both $i = 1$ and $i = 2$, with at least one of the inequalities being strict.

(b) For what values of a_1 and a_2 is the outcome of the Nash equilibrium that you found in part (a) Pareto efficient? Prove your answer formally.

Now assume that there are infinitely many, discrete time periods t (so $t = 1, 2, 3, \ldots$), and at each t the firms simultaneously choose their respective advertising level, $x_{it} \geq 0$. The firms' common discount factor is denoted by $\delta \in [0, 1)$. At the end of each time period, the firms can observe each other's choice of x_{it}. To simplify the model, assume that the efficiency parameters are given by

$$(a_1, a_2) = (2, 3).$$

Let a pair of cooperative advertising levels be given by $(x_1^c, x_2^c) = (4, 6)$. Consider the following grim trigger strategy: In period $t = 1$, each firm i chooses $x_{i1} = x_i^c$. In all later periods t, firm i chooses:

- $x_{it} = x_i^c$ if each firm i chose x_i^c in all previous periods;
- $x_{it} = x_i^n$ if at least one of the two firms chose some $x_{it} \neq x_i^c$ in any previous period.

Here, x_i^n is firm i's Nash equilibrium advertising level in the one-shot version of the game (so the advertising level you were asked to identify in part (a)).

(c) Investigate under what condition the two firms' following the above trigger strategy constitutes a subgame perfect Nash equilibrium of the infinitely repeated game. The condition should be stated as $\delta \geq K$, where K is a particular number (which must be specified).

Question 2: Finding more equilibria in the BBPD model with a mix of naive and sophisticated consumers

This question revisits problem 6.7 ("Behavior-based price discrimination with a mix of naive and sophisticated consumers"), which we studied in the course. The model in problem 6.7 was, in turn, an extension of the simple two-period monopoly model of behavior-based price discrimination (BBPD) that we studied in a problem set. Below you can first find a restatement of the model in problem 6.7.

The model is identical to the BBPD model in the lecture slides (L3-II), except that a fraction $(1-\gamma) \in (0,1)$ of the consumers are *naive*: they buy at a given first-period price p_1 if and only if that price does not exceed the consumer's valuation: $p_1 \leq r$. The remaining fraction of consumers, $\gamma \in (0,1)$, are *sophisticated*: they are, exactly as the consumers in our original model, rational and forward looking and therefore understand that the firm charges different second-period prices (p_2^H and p_2^L) depending on if the consumer bought in the first period or not. The sophisticated consumers take that fact into account when deciding whether to buy in the first period, using a discount factor that equals one: $\delta = 1$.

Within each group there is heterogeneity with respect to the valuation $r \in [0,1]$. The distribution of r's is the same for naive and sophisticated consumers: it is uniform on $[0,1]$. The mass of all consumers (i.e., all naive and all sophisticated consumers together) equals one. The firm is assumed to be myopic, with a discount factor $\beta = 0$. The firm does not have any production costs.

A given consumer's r value is the same across the two periods. The monopoly firm cannot observe this r. Nor can it observe whether an individual consumer is sophisticated or naive. However, the firm knows the fraction of sophisticated consumers in the population (γ) and, in the second period, it knows whether an individual consumer bought in the first period or not.

All in all, the model is identical to the one we studied in the lecture slides (L3-II), except for the presence of naive consumers and the assumptions $\delta = 1$ and $\beta = 0$.

As in the lecture slides, let $\widehat{r}$ denote a cutoff value of r such that sophisticated consumers buy in the first period if and only if $r \geq \widehat{r}$. We will look for an equilibrium of the model that satisfies the following requirements: $p_1 \leq \widehat{r}$ and $\widehat{r} \geq \frac{1}{2}$.

One can show that demand in the second-period "L-market" equals

$$q_2^L = \begin{cases} \gamma\widehat{r} + (1-\gamma)\,p_1 - p_2^L & \text{if } p_2^L \in [0, p_1] \\ \gamma\left(\widehat{r} - p_2^L\right) & \text{if } p_2^L \in [p_1, \widehat{r}] \\ 0 & \text{if } p_2^L \in [\widehat{r}, 1]\,, \end{cases} \tag{2}$$

and therefore profits in the second-period L-market equal $\pi_2^L = q_2^L p_2^L$. One can also show that the price p_2^L that maximizes those profits is given by

$$p_2^L = \begin{cases} \frac{\widehat{r}}{2} & \text{if } p_1 \in \left[0, \frac{\sqrt{\gamma}\widehat{r}}{1+\sqrt{\gamma}}\right] \\ \frac{\gamma\widehat{r}+(1-\gamma)p_1}{2} & \text{if } p_1 \in \left[\frac{\sqrt{\gamma}\widehat{r}}{1+\sqrt{\gamma}}, \widehat{r}\right]. \end{cases} \tag{3}$$

Furthermore one can show that demand in the second-period "H-market" equals

$$q_2^H = \begin{cases} \gamma\,(1-\widehat{r}) + (1-\gamma)\,(1-p_1) & \text{if } p_2^H \in [0, p_1] \\ \gamma\,(1-\widehat{r}) + (1-\gamma)\left(1-p_2^H\right) & \text{if } p_2^H \in [p_1, \widehat{r}] \\ 1-p_2^H & \text{if } p_2^H \in [\widehat{r}, 1]\,, \end{cases} \tag{4}$$

and therefore profits in the second-period H-market equal $\pi_2^H = q_2^H p_2^H$. One can also show that the price p_2^H that maximizes those profits is given by

$$p_2^H = \begin{cases} \widehat{r} & \text{if } \widehat{r} \leq \frac{1}{2-\gamma} \\ \frac{1-\gamma\widehat{r}}{2(1-\gamma)} & \text{if } \widehat{r} > \frac{1}{2-\gamma} \text{ and } p_1 < \frac{1-\gamma\widehat{r}}{2(1-\gamma)} \\ p_1 & \text{if } p_1 \geq \frac{1-\gamma\widehat{r}}{2(1-\gamma)}. \end{cases} \tag{5}$$

In the questions below you will be asked to investigate the possibility of another kind of equilibrium than the one we looked for in problem 6.7. In particular, we will here look for an equilibrium in which the second-period prices are given by the second line of (3) and the *first line* of (5), respectively; that is,

$$p_2^L = \frac{\gamma\widehat{r} + (1-\gamma)\,p_1}{2}, \quad p_2^H = \widehat{r}.$$

(a) By studying the first-period decisions of the sophisticated consumers, derive a relationship between $\widehat{r}$ and p_1 that must hold at an equilibrium.

(b) By studying the firm's profit-maximization problem in period 1, find the firm's optimal choice of p_1. Then use the information about p_1 and the information in (a) to calculate the implied values of $\widehat{r}$, p_2^L and p_2^H. Investigate if these values indeed are part of an equilibrium of the model (if any conditions on the parameters are required, state these).

(c) Derive the expression for q_2^H stated in (4). That is, explain how we can obtain this demand function, given the consumers' preferences and other assumptions that we have made. You are encouraged to use figures, if you think they can help you explain.

End of Exam